Lens-FOV Conversion

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1 Introduction

Usually in 3d graphics, when we set up a perspective projection for a camera, one of the parameters needed is the Field Of View, or FOV. Simple enough, the FOV is the camera's viewing angle and basically, it represents the camera's lens. Real camera lenses however are usually defined by their focal length (a.k.a. focal distance). So what do we do if we need to convert from one to the other?

2 Let's start

The basic idea behind any conventional camera, be it real or virtual, is to project an image from a 3d environment onto a 2d surface - in case of a virtual camera, it would be some render target (back buffer, texture or whatever) and in case of a real camera, it would be some sensor (digital matrix, analog film or whatever). Anyway, we have

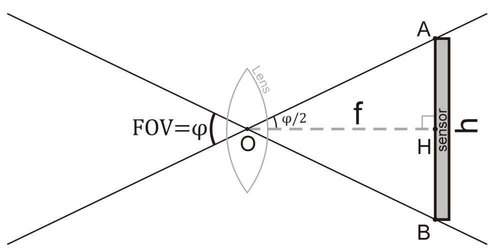

Figure 1: Setting

Let O be the center of the lens
Let $AB = h$ be the sensor
Let $OH = f$ be the focal distance. $H \in AB$ and $OH \perp AB$
$\angle AOB = \varphi = \text{FOV}$

3 Lens to FOV

Given we have

- Focal length f
- Sensor size h

From $\triangle AOH \rightarrow \angle AOH = \arctan \frac{AH}{OH}$
But $\angle AOH = \frac{\varphi}{2}$, $AH = \frac{h}{2}$ and $OH = f \implies \frac{\varphi}{2} = \arctan \frac{h}{2f}$

$$\varphi = 2 \arctan \frac{h}{2f}$$

4 FOV to lens

Given we have

- Field of view φ
- Sensor size h

From $\triangle AOH \rightarrow \tan \angle AOH = \frac{AH}{OH}$
But $\angle AOH = \frac{\varphi}{2}$, $AH = \frac{h}{2}$ and $OH = f \implies \tan \frac{\varphi}{2} = \frac{h}{2f}$

$$f = \frac{h}{2 \tan \frac{\varphi}{2}}$$

5 Sensor size

And so, the conversion is quite simple, with the only thing required in both directions being the size of the sensor/film/whatever. Well, if you have the particular sensor at hand and would like to go cowboy-style at it, you can simply get a ruler and take the measure. Otherwise, you might check for official info on the sensor

An important thing to notice, is that because films have rectangular shape, they'd generally cover different angles along their horizontal size (width), along their vertical size (height) and along their diagonal. Usually, the horizontal is considered, but if you want to be precise, you should check.

5.1 Analog films

There are many standards for analog films, but maybe the most typical among them might be considered the ISO 135 film, which is the type used in most common cameras. It has 36mm width and 24mm height, so you might consider using 36 as a default sensor size.

5.2 Digital sensors

Digital camera sensors usually come by sizes in inches, such as 1", 2/3", 1/2" and et cetera. It is considered for these sensors, that one inch corresponds to diagonal of about 16mm, or about 63% . Because in this case we have the diagonal, if width and height of the sensor are required, they can be calculated with regard of the sensor's aspect ratio, that is
$ratio = \frac{\text{horizontal resolution}}{\text{vertical resolution}} \implies width = height * ratio$
$diagonal^2 = width^2 + height^2 = height^2 * ratio^2 + height^2$
$height = \sqrt{\frac{diagonal^2}{ratio^2+1}}$

5.3 Measuring units

Remember to have the focal length and the sensor size with the same measuring units.

6 Appendix

6.1 3DS Max cameras

Each camera in 3DS Max, cameras can be set which dimension - width, height or diagonal, to fit its FOV. This setting defaults to width.
The size of the sensor there is formed from a settable width, which defaults to 36mm, and a height, which is computed on account of the width and the aspect ratio of the rendering resolution, that is $height = fracwidthaspect$.

6.2 FOV and focal length dependency

6.3 Camera obscura